

Hamiltonian formulation of exactly solvable models and their physical vacuum states

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We clarify a few conceptual problems of quantum field theory on the level of exactly solvable models with fermions. The ultimate goal of our study is to gain a deeper understanding of differences between the usual ("spacelike") and light-front forms of relativistic dynamics. We show that by incorporating operator solutions of the field equations to the canonical formalism the spacelike and light front Hamiltonians of the derivative-coupling model acquire an equivalent structure. The same is true for the massive solvable theory, the Federbush model. In the conventional approach, physical predictions in the two schemes disagree. Moreover, the derivative-coupling model is found to be almost identical to a free theory, in contrast to the conventional canonical treatment. Physical vacuum state of the Thirring model is then obtained by a Bogoliubov transformation as a coherent state quadratic in composite boson operators. To perform the same task in the Federbush model, we derive a massive version of Klaiber's bosonization and show that its light-front form is much simpler.

The usual "spacelike" (SL) and the lightfront (LF) [1] forms of relativistic quantum field theory (QFT) are two independent representations of the same physical reality. There are however striking differences between both schemes already at the level of basic properties [2, 3]. This concerns the mathematical structure as well as some physical aspects (nature of field variables, division of the Poincaré generators into the kinematical and dynamical sets, status of the vacuum state, etc.) Exactly solvable models offer an opportunity to study the structure of the two theoretical frameworks and their relationship since in these models exact operator solutions of field equations are known. From the solutions, essential properties like correlation functions can be computed nonperturbatively and independently of conformal QFT methods [4]. Note that not all solvable models belong to the conformal class. Investigations of their properties in a hamiltonian approach permits us to study directly the role of the vacuum state and of the operator structures in both forms of QFT. Recall that in the LF scheme, Fock vacuum is often the lowest-energy eigenstate of the *full* Hamiltonian. This feature is not present in the SL theory.

In this letter, we give a brief survey of the hamiltonian study of the derivative-coupling model (DCM)[5], the Thirring [6] and the Federbush model (FM) [7]. The unifying idea is to benefit from the knowledge of operator solutions of the field equations to re-express the corresponding Hamiltonians (both in the SL and LF versions) entirely in terms of true degrees of freedom which are the free fields. This previously overlooked aspect not only simplifies the overall physical picture but also removes structural differences between SL and LF Hamilto-

nians. For example, in the case of the simplest theory, the derivative-coupling model, the standard canonical procedure applied to the SL and LF Lagrangians leads to a striking result: the SL Hamiltonian contains an interaction term while its LF analog does not. Making use of exact solutions of the field equations, the SL version of the DCM Hamiltonian is found to have also the interaction-free form. Consequently, the physical SL vacuum coincides with the Fock vacuum in a full agreement with the LF result. However, for truly interacting models, the Fock vacuum is an eigenstate of the free SL Hamiltonians only. Interaction parts of the Hamiltonians are generally non-diagonal when expressed in terms of creation and annihilation operators. To find the true vacuum state, they have to be diagonalized. This is a complicated dynamical problem which however turns out to be tractable analytically for the Thirring and Federbush models. Our approach is to cast their Hamiltonians to the quadratic form by bosonization of the vector current and to diagonalize them by a Bogoliubov transformation, generating thereby the true ground state as a transformed Fock vacuum (a coherent state). We will show this explicitly for the Thirring model. As for the FM, the conventional approach yields a vanishing interaction Hamiltonian for the LF case and a nonvanishing one for the SL case. This discrepancy is removed when the solution of field equations is taken into account leading to interaction Hamiltonians of the same structure. Finally, note that the solvability of the (conformally-noninvariant) massive FM allows one also to test the methods of CFT where the mass term is treated as a perturbation [4].

Derivative-coupling model. It is instructive to ex-

plain our main ideas at a very simple theory – a massive fermion field interacting with a massive scalar field via a gradient coupling. Its Lagrangian and field equations are

$$\begin{aligned}\mathcal{L} = & \bar{\Psi} \left(\frac{i}{2} \gamma^\mu \overset{\leftrightarrow}{\partial}_\mu - m \right) \Psi + \frac{1}{2} (\partial_\mu \phi \partial^\mu \phi - \mu^2 \phi^2) - g \partial_\mu \phi J^\mu, \\ & i \gamma^\mu \partial_\mu \Psi = m \Psi + g \partial_\mu \phi \gamma^\mu \Psi, \\ & \partial_\mu \partial^\mu \phi + \mu^2 \phi = g \partial_\mu J^\mu = 0.\end{aligned}\quad (1)$$

Here $\gamma^0 = \sigma^1, \gamma^1 = i\sigma^2, \alpha^1 = \gamma^5 = \gamma^0 \gamma^1$ and σ^i are the Pauli matrices. $J^\mu(x)$ is the (normal-ordered) vector current. The original Schroer's model [5] had $\mu = 0$. Using the notation $d\tilde{k}^1 = dk^1 / \sqrt{4\pi E(k^1)}, \hat{k} \cdot x = E(k^1)t - k^1 x^1, E(k^1) = \sqrt{k_1^2 + \mu^2}$, the free scalar field, quantized by $[a(k^1), a^\dagger(l^1)] = \delta(k^1 - l^1)$, is expanded as

$$\phi(x) = \int_{-\infty}^{+\infty} d\tilde{k}^1 [a(k^1) e^{-i\hat{k} \cdot x} + a^\dagger(k^1) e^{i\hat{k} \cdot x}]. \quad (2)$$

It enters into the operator solution of the equation (2):

$$\Psi(x) =: e^{ig\phi(x)} : \psi(x), \quad i\gamma^\mu \partial_\mu \psi(x) = m\psi(x). \quad (3)$$

The Fock expansion of the free massive fermion field $\psi(x)$

$$\psi(x) = \int_{-\infty}^{+\infty} d\tilde{p}^1 [b(p^1) u(p^1) e^{-i\hat{p} \cdot x} + d^\dagger(p^1) v(p^1) e^{i\hat{p} \cdot x}] \quad (4)$$

contains the spinors $u^\dagger(p^1) = (\sqrt{p^-}, \sqrt{p^+}), v^\dagger(p^1) = u^\dagger(p^1) \gamma_5, p^\pm = E(p^1) \pm p^1$. The conjugate momenta

$$\Pi_\phi = \partial_0 \phi(x) - gJ^0, \quad \Pi_\Psi = \frac{i}{2} \Psi^\dagger, \quad \Pi_{\Psi^\dagger} = -\frac{i}{2} \Psi \quad (5)$$

lead to the Hamiltonian $H = H_{0B} + H'$, where H_{0B} corresponds to the free massive scalar field and

$$H' = \int_{-\infty}^{+\infty} dx^1 [-i\Psi^\dagger \alpha^1 \partial_1 \Psi + m\Psi^\dagger \gamma^0 \Psi + g\partial_1 \phi J^1].$$

Since the term $(i/2) \bar{\Psi} \gamma^\mu \overset{\leftrightarrow}{\partial}_\mu \Psi$ in the Lagrangian is conventionally taken in terms of the free field, the first term in H' becomes simply $-i\psi^\dagger \alpha^1 \partial_1 \psi$. Setting $m = 0$ for simplicity, the interaction Hamiltonian acquires the form

$$\begin{aligned}H_g = & -\frac{g}{2\sqrt{\pi}} \int_{-\infty}^{+\infty} \frac{dk^1 |k^1|^2}{\sqrt{E(k^1) |k^1|}} [a^\dagger(k^1) c(k^1) + \\ & + c^\dagger(k^1) a(k^1) + a^\dagger(k^1) c^\dagger(-k^1) + a(k^1) c(-k^1)],\end{aligned}\quad (6)$$

where the Klaiber's bosonized representation [8] of the massless vector current was used (see Eq.(14) below). The Hamiltonian (6) is nondiagonal. Its diagonalization can be performed by a Bogoliubov transformation implemented by a unitary operator $U = \exp(iS)$ with

$$iS(\gamma) = \int_{-\infty}^{+\infty} dk^1 \gamma(k^1) [c^\dagger(k^1) a^\dagger(-k^1) - H.c.]. \quad (7)$$

As a result, the real ground state has nontrivial structure, $|\bar{\Omega}\rangle = \exp(-iS(\gamma_d))|0\rangle$ [9] (cf. Eqs.(18-21) below).

All this is true provided the starting Hamiltonian is the correct one. However, this is not the case. The point is that we did not use our knowledge of the operator solution (3) in the course of the derivation of the Hamiltonian (6). In the case of models which are not exactly solvable, one knows $\gamma^\mu \partial_\mu \Psi(x)$ from the corresponding Dirac equation. This expression should not be used in the Lagrangian since the latter would vanish (extremum of the action). In our case, we actually know more, namely $\partial_\mu \Psi(x)$ from the solution (3) and this information (not the free Dirac equation, however!) *should* be used in the starting Lagrangian (2). This is similar to elimination of a nondynamical field by using its constraint in the Lagrangian. Thus, the correct procedure in our case is to insert the solution for the interacting field $\Psi(x)$ into the kinetic term. We find that the interaction cancels completely in the Lagrangian (2) and we get $H = H_{0B} + H_{0F}$,

$$H_{0F} = \int_{-\infty}^{+\infty} dx^1 [-i\psi^\dagger \alpha^1 \partial_1 \psi + m\psi^\dagger \gamma^0 \psi]. \quad (8)$$

Although the full Hamiltonian is simply the sum of free Hamiltonians of the massive scalar and fermion fields and hence the ground state of the DCM is just the Fock vacuum, the model is not completely trivial: one generates the correct Heisenberg equations $i\partial_0 \Psi(x) = -[H, \Psi(x)] = -i\alpha^1 \partial_1 \Psi + m\gamma^0 \Psi - g\partial_0 \phi \Psi - g\alpha^1 \partial_1 \phi \Psi$ with H . Correlation functions computed from the solution (3) will depend on the coupling constant, too. Note also that in the usual treatment the momentum operator contains interaction. This defect is cured in our approach.

The same picture is obtained in the LF analysis. Our notation is $\psi^\dagger = (\psi_1^\dagger, \psi_2^\dagger), x^\mu = (x^+, x^-), \partial_\pm = \partial/\partial x^\pm, \hat{p} \cdot x = 1/2(\hat{p}^- x^+ + p^+ x^-), \hat{p}^- = m^2/p^+, \text{ where } x^+, p^+ \text{ and } j^\mu = (j^+, j^-) \text{ are the LF time, momentum and current. Inserting now the solution (3) of the field equations}$

$$\begin{aligned}2i\partial_+ \Psi_2(x) &= m\Psi_1(x) + 2g\partial_+ \phi(x) \Psi_2(x), \\ 2i\partial_- \Psi_1(x) &= m\Psi_2(x) + 2g\partial_- \phi(x) \Psi_1(x)\end{aligned}\quad (9)$$

into the LF Lagrangian

$$\begin{aligned}\mathcal{L}_{lf} = & 2\partial_+ \phi \partial_- \phi - \frac{1}{2} \mu^2 \phi^2 + i\Psi_2^\dagger \overset{\leftrightarrow}{\partial}_+ \Psi_2 + i\Psi_1^\dagger \overset{\leftrightarrow}{\partial}_- \Psi_1 \\ & - m(\Psi_2^\dagger \Psi_1 + \Psi_1^\dagger \Psi_2) - g\partial_+ \phi j^+ - g\partial_- \phi j^-, \end{aligned}\quad (10)$$

we obtain the Lagrangian of the free LF massive fermion and boson fields with the corresponding LF Hamiltonian

$$P^- = \frac{1}{2} \int_{-\infty}^{+\infty} dx^- [m(\psi_2^\dagger \psi_1 + \psi_1^\dagger \psi_2) + \frac{1}{2} \mu^2 \phi^2]. \quad (11)$$

We recall that in the conventional treatment, one gets a controversial picture: the LF Hamiltonian still remains free while the SL Hamiltonian contains an interaction term (6) and its ground state is the coherent state $|\bar{\Omega}\rangle$.

It is interesting that the analogous model with the axial vector current $j_5^\mu(x)$ is not solvable. One reason is that $j_5^\mu(x)$ is not conserved, and hence the (pseudo)scalar field is not free. More importantly, the naive generalization of the solution (3) to $\Psi(x) =: \exp\{-ig\gamma^5\phi(x)\} : \psi(x)$ actually does not satisfy the corresponding Dirac equation. On the other hand, the Rothe-Stamatescu model [10] ($m = 0$) is exactly solvable but its Hamiltonian is actually a free one, contrary to the claim made in [10].

The Thirring model [6] was extensively studied in the past. A detailed analysis of its operator solution has been made in [8]. A systematic hamiltonian treatment based on the model's solvability was not given so far.

The Lagrangian density of the massless Thirring model and the corresponding field equations read

$$\begin{aligned}\mathcal{L} &= \frac{i}{2}\bar{\Psi}\gamma^\mu \overleftrightarrow{\partial}_\mu \Psi - \frac{1}{2}gJ_\mu J^\mu, \\ i\gamma^\mu \partial_\mu \Psi(x) &= gJ^\mu(x)\gamma_\mu \Psi(x), \\ J^\mu &=: \bar{\Psi}\gamma^\mu \Psi, \quad \partial_\mu J^\mu(x) = 0.\end{aligned}\quad (12)$$

The simplest classical solution is

$$\Psi(x) = e^{-i(g/\sqrt{\pi})j(x)}\psi(x), \quad \gamma^\mu \partial_\mu \psi(x) = 0, \quad (13)$$

implying that the interacting current $J^\mu(x)$ coincides with the free current $j^\mu(x)$. The latter is obtained from the operator $j(x)$ as $\sqrt{\pi}j_\mu(x) = \partial_\mu j(x)$ (see below).

The expansion of the massless spinor field is the $m = 0$ limit of Eq.(4). After the Fourier transformation, the current $j^\mu(x) = (\psi^\dagger(x)\psi(x), \psi^\dagger(x)\alpha^1\psi(x))$ is represented in terms of boson operators:

$$\begin{aligned}j^\mu(x) &= \frac{-i}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{dk^1 k^\mu}{\sqrt{2|k^1|}} \{c(k^1)e^{-i\hat{k}\cdot x} - c^\dagger(k^1)e^{i\hat{k}\cdot x}\}, \\ c(k^1) &= \frac{i}{\sqrt{|k^1|}} \int_{-\infty}^{+\infty} dp^1 \{ \theta(p^1 k^1) [b^\dagger(p^1)b(p^1 + k^1) - \\ &\quad - d^\dagger(p^1)d(p^1 + k^1)] + \\ &\quad + \epsilon(p^1)\theta(p^1(k^1 - p^1))d(k^1 - p^1)b(p^1) \},\end{aligned}\quad (14)$$

with c, c^\dagger obeying the canonical commutation relation,

$$[c(p^1), c^\dagger(q^1)] = \delta(p^1 - q^1), \quad c(k^1)|0\rangle = 0. \quad (15)$$

The Hamiltonian of the model is obtained from the Lagrangian (12) after inserting the solution (13) into it. The interaction is *not* canceled indicating a less trivial dynamics than found in the DC model. However, the contribution of the term $(i/2)\bar{\Psi}\gamma^\mu \overleftrightarrow{\partial}_\mu \Psi$ just reverses the sign of the interaction term leading to

$$H = \int_{-\infty}^{+\infty} dx^1 \left[-i\psi^\dagger \alpha^1 \partial_1 \psi - \frac{1}{2}g(j^0 j^0 - j^1 j^1) \right]. \quad (16)$$

The usual treatment gives $+1/2g$ in Eq.(16) [13, 14]. In the Fock representation, H_0 and H_g have the form

$$\begin{aligned}H_0 &= \int_{-\infty}^{+\infty} dp^1 |p^1| \left[b^\dagger(p^1)b(p^1) + d^\dagger(p^1)d(p^1) \right], \\ H_g &= \frac{g}{\pi} \int_{-\infty}^{+\infty} dk^1 |k^1| \left[c^\dagger(k^1)c^\dagger(-k^1) + c(k^1)c(-k^1) \right].\end{aligned}\quad (17)$$

H_g is not diagonal and thus $|0\rangle$ is not an eigenstate of $H = H_0 + H_g$. To diagonalize H , we form the new Hamiltonians $\hat{H}_0 = H_0 - T$, $\hat{H}_g = H_g + T$ [15], where

$T = \int_{-\infty}^{+\infty} dk^1 |k^1| c^\dagger(k^1)c(k^1)$, and implement a Bogoliubov transformation by the unitary operator $U = e^{iS}$,

$$iS = \frac{1}{2} \int_{-\infty}^{+\infty} dp^1 \gamma(p^1) \left[c^\dagger(p^1)c^\dagger(-p^1) - c(p^1)c(-p^1) \right].$$

\hat{H}_0 is invariant with respect to U . \hat{H}_g transforms non-trivially since $[S, c(k^1)] = i\gamma(k^1)c^\dagger(-k^1)$. This, by $c(k^1) \rightarrow e^{iS}c(k^1)e^{-iS}$, implies

$$c(k^1) \rightarrow c(k^1) \cosh \gamma(k^1) - c^\dagger(-k^1) \sinh \gamma(k^1). \quad (18)$$

The transformed Hamiltonian $e^{iS}\hat{H}_g e^{-iS}$ is diagonal,

$$\hat{H}_g^d = \frac{1}{\cosh 2\gamma_d} \int_{-\infty}^{+\infty} dk^1 |k^1| c^\dagger(k^1)c(k^1), \quad (19)$$

if we set $\gamma(k^1) = \gamma_d = \frac{1}{2} \text{artanh} \frac{g}{\pi}$. Thus we have

$$e^{iS} [\hat{H}_0 + \hat{H}_g] e^{-iS} |0\rangle = 0 \Rightarrow |\Omega\rangle = e^{-iS} |0\rangle. \quad (20)$$

The new vacuum state $|\Omega\rangle$ (where $\kappa = \frac{1}{2} \tanh \gamma_d$),

$$|\Omega\rangle = N \exp \left[-\kappa \int_{-\infty}^{+\infty} dp^1 c^\dagger(p^1)c^\dagger(-p^1) \right] |0\rangle, \quad (21)$$

corresponds to a coherent state of pairs of composite bosons with zero total momentum, $P^1|\Omega\rangle = 0$. The vacuum $|\Omega\rangle$ is invariant under axial $U(1)$ transformations

$$V(\beta)|\Omega\rangle = |\Omega\rangle, \quad V(\beta) = e^{i\beta Q_5},$$

$$Q_5 = \int_{-\infty}^{+\infty} dq^1 \epsilon(q^1) [b^\dagger(q^1)b(q^1) - d^\dagger(q^1)d(q^1)]. \quad (22)$$

Thus, no chiral symmetry breaking occurs. This finding disagrees with the results [13] where a BCS type of ansatz for the vacuum state was used. The true vacuum has to be an eigenstate of the full Hamiltonian, however. $|\Omega\rangle$ is such a state while the BCS-like state is not.

Correlation functions can be calculated from the normal-ordered operator solution [8]

$$\Psi(x) = e^{(-ig/\sqrt{\pi})j^{(+)}(x)}\psi(x)e^{(-ig/\sqrt{\pi})j^{(-)}(x)}, \quad (23)$$

using the vacuum $|\Omega\rangle$. $j^{(\pm)}(x)$ are positive and negative-frequency parts of the integrated current, $j = j^{(+)} + j^{(-)}$,

$$j^{(-)}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dq^1 \frac{c(q^1)}{\sqrt{2|q^1|}} \theta(|q^1| - \lambda) e^{-i\hat{q}\cdot x}. \quad (24)$$

$j^{(+)}(x) = j^{(-)\dagger}(x)$ and η is the conventional infrared cut-off. Further details will be given elsewhere [12].

The Federbush model is defined by the Lagrangian

$$\mathcal{L} = \frac{i}{2} \bar{\Psi} \gamma^\mu \overleftrightarrow{\partial}_\mu \Psi - m \bar{\Psi} \Psi + \frac{i}{2} \bar{\Phi} \gamma^\mu \overleftrightarrow{\partial}_\mu \Phi - \mu \bar{\Phi} \Phi - g \epsilon_{\mu\nu} J^\mu H^\nu, \quad \epsilon^{\mu\nu} = -\epsilon^{\nu\mu}, \quad (25)$$

which describes two species of coupled fermion fields with masses m and μ [7]. Both currents $J^\mu = \bar{\Psi} \gamma^\mu \Psi$, $H^\mu = \bar{\Phi} \gamma^\mu \Phi$ are conserved. The coupled field equations

$$\begin{aligned} i\gamma^\mu \partial_\mu \Psi(x) &= m\Psi(x) + g\epsilon_{\mu\nu} \gamma^\mu H^\nu(x) \Psi(x), \\ i\gamma^\mu \partial_\mu \Phi(x) &= \mu\Phi(x) - g\epsilon_{\mu\nu} \gamma^\mu J^\nu(x) \Phi(x) \end{aligned} \quad (26)$$

are exactly solvable even for non-zero masses:

$$\begin{aligned} \Psi(x) &= e^{-i(g/\sqrt{\pi})h(x)} \psi(x), \quad i\gamma^\mu \partial_\mu \psi(x) = m\psi(x), \\ \Phi(x) &= e^{i(g/\sqrt{\pi})j(x)} \varphi(x), \quad i\gamma^\mu \partial_\mu \varphi(x) = \mu\varphi(x). \end{aligned} \quad (27)$$

In quantum theory, the above exponentials have to be regularized by the "triple-dot ordering" [16]. The potentials $j(x), h(x)$ given in terms of the free currents as $\partial_\mu j(x) = \sqrt{\pi} \epsilon_{\mu\nu} j^\nu(x)$, $\partial_\mu h(x) = \sqrt{\pi} \epsilon_{\mu\nu} h^\nu(x)$ enter into the solutions (27) in an "off-diagonal" way. After inserting the solutions into the Lagrangian (25), the interaction term changes its sign leading to the Hamiltonian

$$H = H_0 + g \int_{-\infty}^{+\infty} dx^1 (j^0 h^1 - j^1 h^0). \quad (28)$$

where H_0 is the sum of two free fermion Hamiltonians.

The LF field equations are also solved by (27) with the free LF fields $\psi(x), \varphi(x)$; $j(x), h(x)$ are given by $2\partial_- j(x) = \sqrt{\pi} j^+(x)$, $2\partial_- h(x) = \frac{1}{2} \sqrt{\pi} h^+(x)$. In the standard LF treatment, one would simply insert the solution of the fermionic constraint into \mathcal{L} . This yields however the free LF Hamiltonian! It is only after inserting the full solution like in the SL case that one obtains the four-fermion interaction term also in the LF case:

$$P_g^- = \frac{1}{2} g \int_{-\infty}^{+\infty} dx^- (j^+ h^- - j^- h^+). \quad (29)$$

The interacting SL Hamiltonian (28) contains terms composed solely from creation or annihilation operators, so

the Fock vacuum is not its eigenstate. The diagonalization can be performed by a Bogoliubov transformation using a *massive* current bosonization. This is considerably more complicated than the massless case [8] but the generalized operators $c(k^1)$ can be derived [12]. The analogous LF operators A, A^\dagger are much simpler and have a structure similar to the massless SL case (14). The bosonized LF current is similar to the SL current (14):

$$j^+(x) = \frac{1}{4\pi} \int_0^\infty \frac{dk^+}{\sqrt{k^+}} k^+ A(k^+, x^+) e^{-\frac{i}{2} k^+ x^-} + H.c. \quad (30)$$

Due to $[A(k^+), A^\dagger(l^+)] = \delta(k^+ - l^+)$, valid at $x^+ = y^+$, the solution (27) can be easily normal-ordered:

$$\begin{aligned} \Phi(x) &= \exp \left\{ -i \frac{g}{\sqrt{\pi}} \hat{A}^\dagger(x) \right\} \exp \left\{ -i \frac{g}{\sqrt{\pi}} \hat{A}(x) \right\} \varphi(x), \\ \hat{A}(x) &= \frac{1}{4\pi} \int_0^\infty \frac{dk^+}{\sqrt{k^+}} A(k^+, x^+) e^{\frac{i}{2} k^+ x^-}. \end{aligned} \quad (31)$$

Similar formulae hold for the solution $\Psi(x)$ built from the operators $B(k^+, x^+), B^\dagger(k^+, x^+)$ which are constructed from $h^+(x)$. The j^- and h^- currents contain the boson operators $C(k^+, x^+), D(k^+, x^+)$ and their conjugates, related to $A, A^\dagger, B, B^\dagger$ via the current conservation. In contrast to its SL analog, the interacting LF Hamiltonian is diagonal and therefore $|0\rangle$ is its lowest-energy eigenstate:

$$\begin{aligned} P_g^- &= \frac{g}{8\pi} \int_0^\infty dk^+ k^+ \left[A^\dagger(k^+) D(k^+) + D^\dagger(k^+) A(k^+) \right. \\ &\quad \left. - B^\dagger(k^+) C(k^+) - C^\dagger(k^+) B(k^+) \right]. \end{aligned} \quad (32)$$

Diagonalization of the bosonized SL Hamiltonian yielding the true SL vacuum state $|\Omega\rangle$ will be given in [12].

The next step will be to compute the correlation functions in both schemes. This task is not simple since one needs to know the commutators of the composite boson operators at unequal times [12]. This is the place where complexities of the usual triple-dot ordering technique [17] enter into our bosonization approach. The knowledge of exact correlation functions will allow us to get a deeper insight into the relation between the SL and LF forms of the Federbush model and of QFT in general.

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